MATHEMATICS 31/31I

CALCULUS REVIEW

PART A:

1. The inflection point for the graph y=f(x) always occurs where

   A. the graph of *f* changes from concave up to concave down (or vice versa)
   B. the first derivative is zero
   C. the second derivative is positive
   D. the second derivative is negative

2. If $y = \frac{1}{\sqrt{1+x}}$ then $\frac{d^2y}{dx^2}$ is

   A. $\frac{-1}{4\sqrt{(1+x)^3}}$
   B. $\frac{3}{4\sqrt{(1+x)^5}}$
   C. $\frac{-1}{2\sqrt{(1+x)^3}}$
   D. $\frac{1}{4\sqrt{(1+x)^3}}$

3. If $y^3 - a^2(x-y) = 0$, $\frac{dy}{dx}$ is

   A. $\frac{a^2}{3y^2 - a^2}$
   B. $\frac{a^2}{3y^2 + a^2}$
   C. $\frac{a^2}{3y^2 + ya^2}$
   D. $\frac{a^2}{3y^2 - ya^2}$

4. The derivative of $y = \frac{x-1}{x(x+1)}$ is

   A. $\frac{-x^2-1}{(x^2+x)^2}$
   B. $\frac{x^2-1}{(x^2+x)^2}$
   C. $\frac{-x^2+2x+1}{(x^2+x)^2}$
   D. $\frac{-x^2-2x+1}{(x^2+x)^2}$

5. If the velocity *v*, in metres per second, of an arrow fired from a bow is given by $v = \frac{300s}{4-2s}$, where *s* is the distance traveled by the arrow, then the acceleration in terms of the distance *s* is given by

A. $\frac{1200(1-s)}{(4-2s)^2}$

B. $\frac{1200(4s-4s^2)}{(4-2s)^2}$

C. $\frac{(300)^2\ 4s}{(4-2s)^3}$

D. none of these

6. The $\lim_{t\to\infty}\left[\frac{2t^2-2t+3}{2t^2+5t-3}\right]$ is equal to

A. 0

B. -1

C. 1

D. $\frac{1}{2}$

7. The $\lim_{h\to 1}\frac{h^2+3h-4}{h-1}$

A. 0

B. -3

C. 5

D. undefined

8. The equation of the tangent to the circle defined by, $x^2+y^2=25$, at the point (3, 4) is

A. $3x-4y-25=0$

B. $3x+4y-25=0$

C. $3x+4y+25=0$

D. $3x-4y+25=0$

9. If $f(x)=x^2$, then the geometric interpretation of $\frac{f(2+a)-f(2)}{a}, a\neq 0$ is the

A. slope of the secant through the points where *x = 2* and *x = 2+a*

B. slope of the tangent through the points where *x = 2* and *x = a*

C. derivative of *y* with respect to *x*

D. equation of the tangent line through the point where *x=2*

10. If $y=u^2-1$ and $x=u^2+1$, the derivative of *y* with respect to *x* is

A. $\frac{-x}{\sqrt{x-1}}$

B. $4u^2$

C. $\frac{x}{x-1}$

D. 1

11. If $y = x^3 - 3px + q$ where p and q are constant and $p>0$ then the value(s) of the function at the stationary point(s) will be

A. maximum only at $x = \sqrt{p}$

B. maximum at $x = -\sqrt{p}$ and minimum at $x = \sqrt{p}$

C. maximum at $x = \sqrt{p}$ and minimum at $x = -\sqrt{p}$

D. nonexistent

12. Given $f(x) = x^2$, then $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ is

A. undefined
B. 0
C. x
D. $2x$

13. Find two natural numbers whose sum is 48 such that the product of the square of one of the numbers times the other number is a maximum.

A. 24 and 24
B. 8 and 40
C. 28 and 20
D. 32 and 16

14. A cylindrical tank has a radius of k feet. It is being filled at the rate of π cubic feet per minute. How fast is the surface rising in feet per minute?

A. πk
B. $(\pi k)^2$
C. $\dfrac{\pi}{k^2}$
D. $\dfrac{1}{k^2}$

15. The hypotenuse of a right isosceles triangle is increasing at 2 cm per minute. The rate of change (cm^2/min) of the area when the hypotenuse is 8 cm is

A. 8
B. 10
C. 12
D. 14

16. A spherical balloon is inflated with gas at the rate of 100 $dm^3 / \min$. Assuming that the gas pressure remains constant then the rate, in *dm* per minute, at which the radius of the balloon is increasing at the instant the radius of the balloon is *3 dm* is

A. $\dfrac{25}{9\pi}$
B. $\dfrac{9\pi}{25}$
C. 25π
D. 9π

17. The product of two positive numbers is 16. If the sum of one number and the square of the other is a minimum, then one of the number is

A. 2
B. 4
C. 6
D. 16

18. The greatest amount by which a number can exceed it square is

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. $\frac{3}{4}$
D. 1

19. If $y = F(u)$ where $u = f(x)$ then the result $\left(\frac{dy}{dx}\right) = \frac{dy}{du} \bullet \frac{du}{dx}$ is best described by

A. chain rule for derivatives
B. composite function derivatives
C. product law for derivatives
D. quotient law for derivatives

20. The derivative of $\frac{x^3 - 7}{x^2 + 4}$ is

A. $\frac{5x^4 + 12x^2 - 14x}{(x^2 + 4)^2}$
B. $\frac{x^4 + 12x^2 + 14x}{(x^2 + 4)^2}$
C. $\frac{-x^4 + 12x^2 - 14x}{(x^2 + 4)^2}$
D. none of these

21. For $x^2 y - 3y^2 = 7$, $\frac{dy}{dx}$ is

A. $\frac{6y - x^2}{2xy}$
B. $\frac{7 - 2xy}{x^2 - 6y}$
C. $\frac{-2xy}{x^2 - 6y}$
D. $\frac{7 - 6y - x^2}{2xy}$

22. The intervals in which the function defined by $y = x^2 - \frac{1}{2}x^4$ is decreasing is

A. $0 < x < 1$ *or* $x < -1$
B. $x < 0$ *or* $x > 1$
C. $x > 0$ *or* $x < -1$
D. $-1 < x < 0$ *or* $x > 1$

23. If a rocket travels t^3 m the first *t* seconds after take off, then its velocity in m/sec when it reaches, 1 000 m is

A. 1000
B. 500
C. 300
D. 100

24. If the height of a particle at any time *t* is given by $h = t^3 + 4t^2$ then the acceleration is given by

A. $6t + 8$
B. $\frac{1}{4}t^4 + \frac{4}{3}t^3$
C. $3t^2 + 8t$
D. $32t$

25. If the sum of an infinite geometric series is 128 and the first term is 16, the common ratio is

A. $\frac{1}{8}$
B. $\frac{7}{8}$
C. $\frac{8}{9}$
D. $\frac{9}{8}$

26. $\lim_{x \to 1} \frac{2x^2 - 6x + 4}{x^2 - 1}$ is equal to

A. 2
B. 1
C. 0
D. -1

27. The slope of the secant of the graph of $y = 4x - x^3$ through the points where $x = 0$ and $x = 1$ is

A. 1
B. 3
C. $\frac{1}{3}$
D. -3

28. A car is driven for 3 hours at 45 km/h and then for 5 hours at 65 km/h. It's average velocity in km/h over the total distance is

A. 55
B. 57.7
C. 57
D. 60

29. If $S_n = \frac{a(1-r^n)}{1-r}$, the sum of the infinite geometric series $3+1=\frac{1}{3}+\frac{1}{9}+\ldots..$ is

A. 5
C. $\frac{40}{9}$
B. $\frac{9}{2}$
D. ∞

30. The equation of the tangent to the curve of $y = 2x^2 + 3x - 5$ at the point where $x = -1$ is

A. $y = 4x + 3$
C. $y = \frac{3}{4}x - \frac{27}{4}$
B. $y = -x - 7$
D. $y = 2x - 2$

31. A point where the graph $y = x^3 - 4x^2 + 4x - 3$ has zero is

A. (2, -3)
C. (-3, 2)
B. (2, 0)
D. `(0, -3)

32. The maximum value of the position given by $s = 3 - 9t + 6t^2 - t^3$ when $t \geq 2$ is

A. -1
C. 1
B. 6
D. 3

33. $\lim_{x \to 0}\left[\frac{3x^3 - 2x^2 + 5x}{x^2 - x}\right]$ is

A. 6
C. 0
B. -5
D. ∞

34. The derivative with respect to x of $y = x\sqrt{1-x^2}$ is

A. $\frac{1}{2}x(1-x^2)^{\frac{-1}{2}} + (1-x^2)^{\frac{1}{2}}$
B. $-x^2(1-x^2)^{\frac{-1}{2}} + (1-x^2)^{\frac{1}{2}}$
C. $-x(1-x^2)^{\frac{-1}{2}}$
D. $x - x(1-x^2)^{\frac{3}{2}}$

35. The derivative of y with respect to x for a function determined by $2x^2+3xy-y^2=20$ is

A. $\dfrac{-(4x+3y)}{(3x-2y)}$ C. $4x+3-2y$

B. $\dfrac{4x+3y}{3x-2y}$ D. $-7x-y$

36. An equation of a tangent to the graph of $y=3x+\dfrac{1}{x}$ which is parallel to the graph of $y=-x+6$ is

A. $y=3-\dfrac{1}{x^2}$ C. $y=-x+4$

B. $y=-x+7\sqrt{\dfrac{1}{3}}$ D. $y=x+4$

37. A local maximum value of $y=x^3+3+\dfrac{3}{x}$ is

A. 7 C. -1

B. 3 D. 1

38. A rectangular field is to be enclosed by a fence and divided into two smaller plots by a fence parallel to one of the sides. The dimensions in feet of the largest such field if 1200 feet of fence is available is

A. 150 x 350 C. 300 x 300

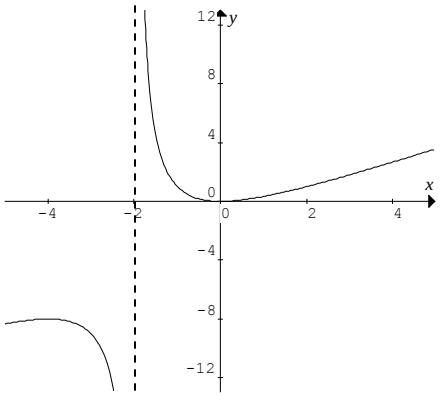
B. 200 x 600 D. 200 x 300

39 A point of inflection on the graph of $y=2x^{\frac{1}{3}}+x^{\frac{4}{3}}$ is

A. (0, 0) C. (1, 3)

B. (8, 20) D. there is no point of inflection

40. The equation of this graph is



A. $y=\dfrac{x^2}{x+2}$ C. $y=\dfrac{x}{x^2-4}$

B. $y=\dfrac{x^2}{(x+2)^2}$ D. $y=\dfrac{x}{x+2}+x$

41. A cube is expanding in such a way that its edge is changing at a rate of 5 cm/s. When its edge is 4 cm long, the rate of change of its volume in cm^3/s is

A. 192 C. 375
B. 48 D. 240

Part B

1. The general solution of $\frac{dy}{dx} = x^n,\ n \neq -1$, is

A. $y = nx^{n-1}$ C. $y = \frac{x^{n+1}}{n+1}$
B. $y = \frac{x^{n+1}}{n+1} + c$ D. $y = x^{n+1} + c$

2. Given the acceleration of a particle as $a = (t^2+1)^2$, then the position *s*, (where c and c_1 are constants), is

A. $\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + c$ C. $\frac{1}{30}t^6 + \frac{1}{6}t^4 + \frac{1}{2}t^2 + ct + c_1$
B. $\frac{1}{30}t^6 + \frac{1}{6}t^4 + \frac{1}{2}t^2 + ct$ D. $\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + c + c_1$

3. The antiderivative of $f(x) = (2x^2-1)^2 x$ is

A. $(2x^2-1)^3 + c$ C. $(2x^2-1)^3$
B. $\left(\frac{2x^2-1^3}{6}\right)$ D. $\frac{(2x^2-1)^3}{12} + c$

4. The area of the region bonded by the graph $y = x^2 + 5$, the *x –axis*, and the lines defined by $x = -2$ and $x = 0$ is

A. $7\frac{1}{3}$ C. $12\frac{2}{3}$
B. 13 D. 18

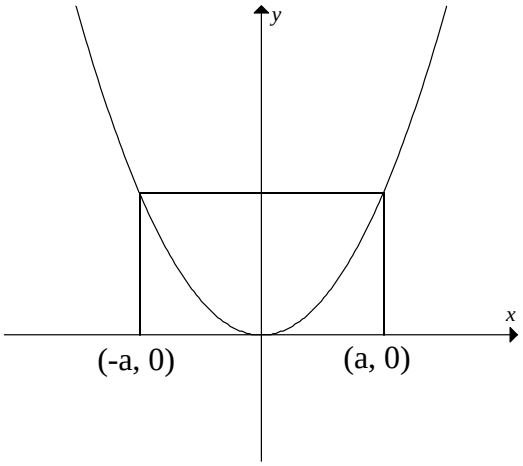
5. Find the area of the region bounded by the graphs of $y = 2x + 4$ and $y = x^2 + 2x$

A. $\frac{20}{3}$ C. $\frac{4}{3}$
B. $\frac{28}{3}$ D. $\frac{32}{3}$

6. The figure at the right shows a rectangle with two vertices at (-a, 0) and (a, 0) and the other two on the parabola $y=3x^2$. What fraction of the area of the rectangle lies below the parabola?

A. $\frac{1}{6}$ C. $\frac{2}{3}$

B. $\frac{1}{3}$ D. $\frac{1}{2}$



7. The general solution of $\frac{dy}{dx} = \frac{x}{\sqrt{2x^2+1}}$ is

A. $\frac{1}{2}\sqrt{2x^2+1}+k$ C. $\frac{-1}{6\sqrt{2x^2+1)^3}}+k$

B. $\frac{1}{6}\sqrt{(2x^2+1)^2}+k$ D. $2x\sqrt{2x^2+1}+k$

8. The general solution of $\frac{dy}{dx} = \frac{3x^2}{(x^3+1)^3}$ is

A. $\frac{1}{2(x^3+1)^2}+c$ C. $\frac{4x^3}{(x^3+1)^4}+c$

B. $\frac{-1}{2(x^3+1)^2}+c$ D. $\frac{-4x^3}{(x^3+1)^4}+c$

9. The area between the graph of $y=4x,\ y=3x$ and the line $x=-2$ is

A. 8 C. 4

B. 6 D. 2

10. The area in square units between the graph of $y = 9 - x^2$, the *x*-axis, and the lines *x=3* and *x=-2* is

A. $56\frac{2}{3}$ C. 10

B. $33\frac{1}{3}$ D. $\frac{8}{3}$

11. The velocity of an object in m/s is given by $v = (t+1)^{\frac{1}{2}}$. The position s in m at any time *t* in seconds if *s=1* when *t=0* is

A. $s = \frac{2}{3}(t+1)^{3/2} + \frac{1}{3}$ C. $s = \frac{3}{2}(t+1)^{3/2} - \frac{1}{2}$

B. $s = \frac{1}{2}(t+1)^{-1/2} + \frac{1}{2}$ D. $s = \frac{1}{2}(t+1)^{-1/2} - \frac{1}{2}$

12. The area in square units enclosed by the curves defined by $y = x^2 - 2x$ and *y=x* is

A. $\frac{4}{3}$ C. $\frac{11}{6}$

B. $\frac{43}{6}$ D. $\frac{9}{2}$

13. A golf ball is putted on a level green with an initial velocity of *20 dm/s*. Due to friction the velocity decreases at the rate of 8 dm/s^2. The distance the ball travel in *dm* is

A. 75 C. 25

B. 45 D. 15

14. The family of curves which has a slope given by $x^{\frac{1}{2}} + 2$ is

A. $y = \frac{x^{\frac{-1}{2}}}{2} + 2x + c$ C. $y = \frac{x^{\frac{-1}{2}}}{2} + c$

B. $y = x^{\frac{1}{2}} + c$ D. $y = \frac{2x^{\frac{3}{2}}}{3} + 2x + c$

15. The value of *y*, if $\frac{dy}{dx} = 2x^3(x+5)$, is

A. $\frac{x^4}{2}\left(\frac{x^3}{2}+5x\right)+c$

C. $\frac{2}{5}x^5+\frac{5}{2}x^4+c$

B. $8x^3+30x^2+c$

D. none of these

16. The area in square units of the region bounded by the graph of $y=\frac{1}{x^3}$, the *x-axis* and the lines with equations *x = 1* and *x = 3* is

A. 240

C. $\frac{26}{27}$

B. $\frac{4}{9}$

D. 20

17. The formula for the position *s* in terms of time *t*, if $v=2-t^{-2}$ and $s=5$ when $t=1$ is

A. $s=2t+t^{-1}$

C. $s=2t^{-1}+5$

B. $s=2t+\frac{1}{t}+2$

D. $s=2t^{-3}+3$

18. The area in square units of the region enclosed by the graphs of $y=x^2$ and $y=x+6$ is

A. $\frac{35}{3}$

C. $\frac{125}{6}$

B. $\frac{215}{3}$

D. $\frac{265}{6}$

19. A truck rolls down a slope inclined 30^o to the horizontal so that its constant acceleration due to gravity is 32 sin 30^o. If the truck starts to roll from rest its speed in dm/s when it has rolled 1000 dm is

A. $5\sqrt{5}$

C. $80\sqrt{10}$

B. $80\sqrt{5}$

D. 120

20. The equation of the curve that has a slope given by $x^2-4x+1$and passes through the point (0, 3) is

A. $y=x^3-6x^2+3x+9$

C. $y=\frac{1}{3}x^3-2x^2+x+1$

B. $y=x^3-6x^2+3x+3$

D. $y=\frac{1}{3}x^3-2x^2+x+3$

ANSWER KEY:

Part A

1.	**A**	**11.**	**B**	**21**	**C**	**31.**	**A**	**41.**	**D**
2.	**B**	**12.**	**D**	**22.**	**D**	**32.**	**D**		
3.	**B**	**13.**	**D**	**23.**	**C**	**33.**	**B**		
4.	**C**	**14.**	**D**	**24.**	**A**	**34.**	**B**		
5.	**C**	**15.**	**A**	**25.**	**B**	**35.**	**A**		
6.	**D**	**16.**	**A**	**26.**	**D**	**36.**	**C**		
7.	**C**	**17.**	**A**	**27.**	**B**	**37.**	**C**		
8.	**B**	**18.**	**A**	**28.**	**B**	**38.**	**D**		
9.	**A**	**19.**	**A**	**29.**	**B**	**39.**	**C**		
10.	**D**	**20.**	**B**	**30.**	**B**	**40.**	**A**		

Part B

1.	**B**	**11.**	**A**
2.	**C**	**12.**	**D**
3.	**D**	**13.**	**C**
4.	**C**	**14.**	**D**
5.	**D**	**15.**	**C**
6.	**B**	**16.**	**B**
7.	**A**	**17.**	**B**
8.	**B**	**18.**	**C**
9.	**D**	**19.**	**B**
10.	**B**	**20.**	**D**