Math 31 Mock Review Exam A

Part A Multiple Choice

1. If $a \neq 0$, then $\lim_{x \to a} \frac{x^2 - a^2}{x^4 - a^4}$ is

   A. $\frac{1}{a^2}$

   B. $\frac{1}{2a^2}$ *

   C. $\frac{1}{6a^2}$

   D. 0

2. $\lim_{x \to 0} \frac{|x|}{x}$ is

   A. −1
   B. 0
   C. 1
   D. nonexistent*

3. If $f(x) = \sqrt[5]{x}$, then $f'(3)$ is

   A. $\frac{1}{(5(3))^{5/4}}$

   B. $\frac{1}{5\left(\sqrt[5]{27}\right)}$

   C. $\frac{\sqrt{3}}{5}$

   D. $\frac{1}{5\left(\sqrt[5]{3^4}\right)}$ *

4. If $f(x)=(x+1)^{3/2}-e^{x^2-9}$, then $f'(3)$ is

A. -5
B. -3 *
C. 1
D. 3

5. If $s(x)=\sin^2 x$, then $s''(x)$ is

A. 2
B. $-2\cos x\sin x$
C. $2\sin x\cos x$
D. $2\cos^2 x-2\sin^2 x$ *

6. The equation of the tangent to the curve of $y=2x^2+3x-5$ at the point where $x=-1$ is

A. $y=4x+3$
B. $y=-x-7$ *
C. $y=\frac{3}{4}x-\frac{27}{4}$
D. $y=2x-2$

7. The slope of the curve $f(x)=3x\sqrt{2x^2+1}$ at the point $x=2$ is

A. 6
B. 17*
C. 18
D. 27

8. If the velocity *v*, in metres per second, of an arrow fired from a bow is given by $v=\frac{300s}{4-2s}$, where *s* is the distance traveled by the arrow, then the acceleration in terms of the distance *s* is given by

A. $\frac{1200(1-s)}{(4-2s)^2}$
B. $\frac{1200(4s-4s^2)}{(4-2s)^2}$
C. $\frac{(300)^2\ 4s}{(4-2s)^3}$ *
D. none of these

9. A cylindrical tank has a radius of k m. It is being filled at the rate of π cubic feet per minute. How fast is the surface rising in metres/minute?

A. πk

B $\dfrac{\pi}{k^2}$

C. $(\pi k)^2$

D. $\dfrac{1}{k^2}$ *

10. A cube is expanding in such a way that its edge is changing at a rate of 5 cm/s. When its edge is 4 cm long, the rate of change of its volume in cm^3/s is

A. 192
B. 375
C. 240*
D. 48

11 If $f(x)=\dfrac{16-4x}{(x+2)(x-3)(x-1)}$, then the asymptotes of $f(x)$ are

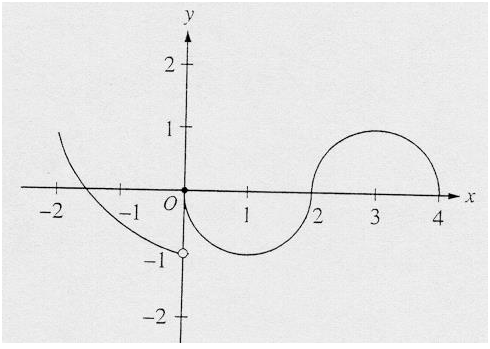
A. $x=-3, x=-1, x=2, y=0$
B. $x=-2, x=1, x=3, y=0$ *
C. $x=-2, x=1, x=3, y=4$
D. $x=-3, x=-1, x=2, y=4$

12 If $f(x)=8x^{1/3}+x^{4/3}$, the interval for which the graph of f is concaved down is

A. $(2,4)$
B. $(0,4)$ *
C. $(-\infty,0)$
D. $(-\infty,2)$

13 The graph of the function f shown in the figure below has a vertical tangent at the point $(2,0)$ and horizontal tangents at $(1,-1)$ and $(3,1)$. For what values of x, $-2<x<4$, is f not differentiable?

A. 0 only
B. 0 and 2 only *
C. 1 and 3 only
D. 0, 1 and 3 only



14 The maximum area of a right triangle with hypotenuse 7 is

A. 10.25
B. 12.25*
C. 15.75
D. 20.5

15 A rectangular field is to be enclosed by a fence and divided into two smaller plots by a fence parallel to one of the sides. The dimensions in metres of the largest such field if 1200 m of fence is available is

A. 150×350
B. 300×300
C. 200×600
D. 200×300 *

16. The product of two positive numbers is 16. If the sum of one number and the square of the other is a minimum, then one of the numbers is

A. 2*
B. 4
C. 6
D. 16

17. The area of the region bounded by the graph $y = x^2 + 5$, the x–axis, and the lines defined by $x = -2$ and $x = 0$ is

A. $7\frac{1}{3}$
B. 13
C. $12\frac{2}{3}$ *
D. 18

18. The family of curves which has a slope given by $x^{\frac{1}{2}}+2$ is

A. $y=\frac{x^{\frac{-1}{2}}}{2}+2x+c$

B. $y=x^{\frac{1}{2}}+c$

C. $y=\frac{x^{\frac{-1}{2}}}{2}+c$

D. $y=\frac{2x^{\frac{3}{2}}}{3}+2x+c$ *

19. $\int_0^{\pi/4}\sin x dx-\int_{-\pi}^{\pi}\cos dx$

A. $\frac{1-\sqrt{2}}{2}$

B. $\frac{2-\sqrt{2}}{2}$ *

C. $\frac{1+\sqrt{2}}{2}$

D. $\frac{2+\sqrt{2}}{2}$

20. $\int_0^{\pi/6}\cos x\left(\sin^2 x+1\right)dx$

A. $\frac{13}{24}$ *

B. $\frac{13\sqrt{3}}{24}$

C. $\frac{-13}{24}$

D. $-\frac{13\sqrt{3}}{24}$

Part B Fill in the blanks

1. The value of $\lim_{x\to 0}\dfrac{\cos x-1}{\sin x}$ is _____ (0)

2. If $f(x)=\log_3(x^2-4)$, then $f'(3)$, to the nearest hundredth, is _____ (1.09)

3. The point(s) of inflection where the function e^{1-x^2} is concave down is(are) _____. $\left(\pm\dfrac{1}{\sqrt{2}}, e^{1/2}\right)$

4. If $\begin{cases} f(x)=\dfrac{x^2-x}{2x} \text{ for } x\neq 0 \\ f(0)=k \end{cases}$

   and if f is continuous at $x=0$, then k is _____ $\left(-\dfrac{1}{2}\right)$

5. If $f(x)=xe^{-x}$, the point of inflection is _____ $\left(2,\dfrac{2}{e^2}\right)$

Part C Written Response

1. Find the derivative of the given function using the **limit definition :** $\dfrac{dy}{dx}=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

   $f(x)=\dfrac{1}{x-2}$ $\left(f'(x)=-\dfrac{1}{(x-2)^2}\right)$

2. Find the equation of the tangent line to the curve $y = \ln\left(x^2 + 1\right)$ when $x = 3.\left(\frac{3}{5}\right)$

3. A cylindrical can holds 1000 cm^3 of oil. Find the radius of the can with minimum surface area. $\left(5.4 \text{ cm}\right)$

4. A ball is thrown vertically upwards into the air. The height, ***h*** meters, of the ball above the ground after ***t*** seconds is given by: $h = 20t - 5t^2, t \geq 0$

   a. find the initial height above the ground. (0)

   b. at what time will the height be 15 meters $\left(t = 1, 3\right)$

   c. when does the ball reach its maximum height? $\left(t = 2\right)$

   d. what is the maximum height of the ball? $\left(20 \text{ m}\right)$

   e. how long is the object in the air? $\left(t = 4\right)$

   f. with what velocity does it hit the ground? $\left(-20 \text{ m/s}\right)$