Math 31 Mock Review Exam B

Part A Multiple Choice

1. The value of the $\lim_{x\to\infty} \dfrac{x^2-4}{2+x-4x^2}$ is

   A. -2
   B. $-\dfrac{1}{4}$ *
   C. $\dfrac{1}{2}$
   D. 1

2. The value of the $\lim_{x\to 0} \dfrac{\sin 2x}{2x}$

   A. -1
   B. -2
   C. 1*
   D. $\dfrac{1}{2}$

3. Given $h(x)\begin{cases} x^2-1 & \text{if } x<0 \\ x-1 & \text{if } 0\le x\le 3 \\ x^3 & \text{if } x>3 \end{cases}$, then h is discontinuous at

   A. $x=0$
   B. $x=0$ and $x=3$
   C. $x=3$ *
   D. $x=\pm 1$

4. If $g(x)=x\left(2^x\right)$, then $g'(x)$ is

   A. $2^x \ln 2$
   B. $2^x\left(x\ln x\right)$
   C. $2^x\left(x\ln 2+1\right)$ *
   D. $2^x\left(x\ln 2\right)$

5. If $y = \frac{1}{\sqrt{1+x}}$ then $\frac{d^2y}{dx^2}$ is

A. $\frac{-1}{4\sqrt{(1+x)^3}}$

B. $\frac{3}{4\sqrt{(1+x)^5}}$ *

C. $\frac{-1}{2\sqrt{(1+x)^3}}$

D. $\frac{1}{4\sqrt{(1+x)^3}}$

6. If $f(x) = \frac{e^{2x}}{\sin x}$ then $f'(x)$ is

A. $\frac{e^{2x}(\cos x - 2\sin x)}{\sin^2 x}$

B. $\frac{e^{2x}(2\cos x - \sin x)}{\sin^2 x}$

C. $\frac{e^{2x}(\sin x - 2\cos x)}{\sin^2 x}$

D. $\frac{e^{2x}(2\sin x - \cos x)}{\sin^2 x}$ *

7. The derivative, $\frac{dy}{dx}$, of $y = \frac{(4x-3)^2}{\sqrt{x}}$ is

A. $\frac{3(4x-3)}{2x\sqrt{x}}$

B. $\frac{3(4x-1)(4x+3)}{2x^{3/2}}$

C. $\frac{3(4x-3)(4x+1)}{2x^{3/2}}$ *

D. $\frac{9(4x-3)}{2\sqrt{x}}$

8. The slope of $9x - 4x\ln y = 3$ at $\left(\frac{1}{3}, 1\right)$ is

A. $9 - 4\ln 3$
B. $9 + 4\ln 3$
C. 6
D. $\frac{27}{4}$ *

9 The slope of the tangent line to the curve $y = \cos^2(3x)$ at the point $x = \frac{\pi}{4}$ is

A. -3
B. $1/2$
C. 2
D. 3*

10. For the motion of a particle on a straight line is given by $s = t^3 - 6t^2 + 12t - 8$, the distance *s* is increasing for

A. $t < 2$
B. all t except $t = 2$*
C. $t > 2$
D. $1 < t < 3$

11. For the motion of a particle on a straight line is given by $s = t^4 - 6t^3 + 12t^2 + 3$, the particle is at rest when *t* is equal to

A. 1 or 2
B. 0*
C. $\frac{9}{4}$
D. 0,2, or 3

12. The hypotenuse of a right isosceles triangle is increasing at 2 cm per minute. The rate of change (cm^2/min) of the area when the hypotenuse is 8 cm is

A. 8*
B. 10
C. 12
D. 14

13. The equation of a slant asymptote of $y = \frac{-2x^2 + x + 4}{-x+1}$ is

A. $y = 2x - 1$
B. $y = 2x + 1$*
C. $y = x + 4$
D. $y = x - 1$

14. A local maximum value of $y = x^3 + 3 + \frac{3}{x}$ is

A. 7
B. 3
C. -1*
D. 1

15. The point of inflection for $y = 2x^2 + \sin 2x$ is

A. $x = \frac{\pi}{4}$ *

B. $x = -\frac{\pi}{4}$

C. $x = \frac{\pi}{3}$

D. $x = -\frac{\pi}{3}$

16. $\int \frac{\ln\sqrt{x}}{x}\, dx$

A. $\frac{1}{4}(\ln x)^2 + C$ *

B. $(\ln x)^2 + C$

C. $\frac{1}{2}\ln|\ln x| + C$

D. $\frac{(\ln\sqrt{x})^2}{2} + C$

17. The area in square units between the graph of $y = 9 - x^2$, the x-axis, and the lines $x = 3$ and $x = -2$.

A. $56\frac{2}{3}$

B. $33\frac{1}{3}$ *

C. 10

D. $\frac{8}{3}$

18. The value of y, if $\frac{dy}{dx} = 2x^3(x+5)$, is

A. $\frac{x^4}{2}\left(\frac{x^3}{2} + 5x\right) + C$

B. $8x^3 + 30x^2 + C$

C. $\frac{2}{5}x^5 + \frac{5}{2}x^4 + C$ *

D. nonexistent

19. The general solution of $\frac{dy}{dx} = \frac{3x^2}{(x^3+1)^3}$ is

A. $\frac{1}{2(x^3+1)^2} + C$

B. $\frac{-1}{2(x^3+1)^2} + C$ *

C. $\frac{4x^3}{(x^3+1)^4} + C$

D. $\frac{-4x^3}{(x^3+1)^4} + C$

20. $\int x^3\left(x-\sqrt{x}+2\right)dx$

A. $4x^3-\left(\frac{7}{2}\right)x^{5/2}+6x^2+C$

B. $\frac{x^4}{4}\left(\frac{x^2}{2}-\frac{2}{3}x^{3/2}+2x\right)+C$

C. $\frac{x^5}{5}-\frac{2}{9}x^{9/2}+\frac{x^4}{2}+C$ *

D. $3x^2\left(1-\frac{1}{2\sqrt{x}}\right)+C$

Part B Fill in the blanks

1. The derivative, $\frac{d}{dx}\left(e^{\ln\left(x^3+7x^2+6\right)}\right)$ is ________________ $\left(3x^2+14x\right)$

2. The value of $\ln 2+\ln 5-\ln 8-\ln 15$ is __________ $\left(-\ln 12\right)$

3. Use the method of separation of variables to solve the differential equation $\frac{dy}{dx}=y\sqrt{x}$

______________________ $\left(\ln|y|=\frac{2x^{3/2}}{3}+C\right)$

4. The interval that the function $\frac{e^x}{x+1}$ is decreasing is________________ $(-\infty,0)$

5. The interval that the function $y=\sin^2 x-\frac{x}{2}$ for $0\le x\le 2\pi$ is increasing is __________

$\left(\frac{\pi}{12},\frac{5\pi}{12}\right)\cup\left(\frac{13\pi}{12},\frac{17\pi}{12}\right)$

Part C Written Response

1. Find the derivative of the given function using the **limit definition :** $\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$f(x)=\sqrt{x+1}$ $\left(f'(x)=\frac{1}{2\sqrt{x+1}}\right)$

2. A man 2 m tall walks away from a lamppost whose light is 5 m above the ground. If he walks at a speed of 1.5 m/s, at what rate is his shadow growing when he is 10 m from the lamppost? $(\ 1\text{m/s})$

3. A toy tugboat is launched from the side of a pond and travels north at 5 cm/s. At the same moment, a toy yacht begins from a point 800 cm east of the tugboat and travels west at 7 cm/s. How closely do the two boats approach each other? (465 cm)

4. A ball is tossed upward on the planet Marwayne, where acceleration due to gravity is 8 m/s^2. The ball is tossed from a height of 1.5 m at an initial velocity of 12 m/s. When will the ball land? (3.12 s)